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algorithmic mathematics and mathematical modeling

COMPETITIVENESS FUNCTION FOR A BILINGUAL COMMUNITY MODEL

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Abstract

The purpose of this work is to construct competitiveness functions for the bilingual community model.

Materials and methods. The work uses a new model of a bilingual community, which takes into account: the effect of acquiring a second language at an early age; the effect of mutual assistance within a group of the same language. In the model, languages are characterized by parameters of prestige, the likelihood of language acquisition at an early age, the parameter of mutual assistance and the initial number of native speakers. The problem of determining the results of language competition based on their characteristic parameters is considered.

Results. A new method for solving the problem of the results of language competition is proposed. For this purpose, a new concept is introduced in linguistic dynamics: the competitiveness function. To restore the competitiveness function, a ranking method is used, which is related to dividing ordered pairs of languages (under fixed initial conditions) into two classes “the first language displaces the second” and “the second language displaces the first”. The competitiveness function is sought in the form of a power function depending on the language parameters. In this case, the values of the function coefficients are identified based on the processing of available data on the dynamics of the model. The values of the competitiveness functions are analyzed, the results are compared with the observed statistics, and on this basis a forecast is made for the further development of dynamics. The application of this technique is demonstrated on a model in which finding a solution in analytical form is difficult.

Conclusion. The proposed methodology for constructing the competitiveness function is quite general and can be applied to a wide range of models describing population dynamics. The forecast made on the basis of the constructed competitiveness functions agrees well with empirical data.

Keywords: *Language competition; language dynamics; bilingualism; selection; language preservation; competitiveness function; selection criterion; selection processes; mathematical model; ordinary differential equations.*

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1. INTRODUCTION

Mathematical modeling is widely used to study language dynamics [1–7]. Abrams and Strogatti laid the foundation for mathematical modeling of language dynamics. They proposed a simple model of language, which allows us to explain historical data on the decline of Welsh, Scottish, Gaelic and other endangered languages [1]. The Abrams-Strogatti (AS) model assumes a homogeneous population, all members of which speak one of two languages. This hypothesis assumes that even a member of a community who knows two languages prefers only one in life, and we can talk not about language proficiency, but about its actualization at a given point in time. Abrams and Strogatti introduced the concepts of language prestige, its attractiveness to those do not speak it, and language volatility as the readiness of native speakers to change it. The number of community members is assumed to be constant. The AS model shows that one language is always replaced by another over time. The latter in the AS model of language dynamics is called language death [1].

When studying language competition, the most important thing is not the competitive dynamics deployed in time, but its result (selection). The presence of such a result makes it possible to identify trends in which one or another language will displace others or disappear. Since the concept of language death is a general and critical trend in language dynamics, the central task for the researcher is to identify the reasons why one language displaces the others, as well as to find solutions to change it. The forecast is the result of selection, which is determined on the basis of the characteristics of the language and the initial number of its speakers.

From the point of view of qualitative analysis of a dynamic system, selection is the movement of a phase trajectory towards a certain state of equilibrium located on the coordinate axis. In this case, the phase space is divided into basins of attraction of stable equilibrium states. Each basin of attraction corresponds to the survival of one language. To predict the results of selection, it is necessary to express the equations of the boundaries separating the basins of attraction. In some models, this can be done by the classical method of studying dynamic systems, but often in models with complex dynamics, determining the basins of attraction becomes a much more complex and non-trivial mathematical task, because the analytical expression of the boundaries separating basins of attraction is very difficult. We propose to do this by constructing a competitiveness function for each language. Comparing the values of the competitiveness function for languages allows us to decide which language will be displaced or remain.

The competitiveness function is understood as a function that depends on the parameters of the language: mutual assistance, likelihood of language acquisition at an early age, prestige and number of native speakers at the initial point in time. If the 1st language eventually displaces the 2nd language, then the value of the competitiveness function for the 1st language should be greater than the value for the 2nd language. Restoring the competitiveness function is a special case of the more general problem of finding comparison functions in ranking problems. In some cases, it is constructed analytically [8–19], but for the model we are considering, the classical research method does not allow us to explicitly express the competitiveness function, so we have to use numerical methods [20**?** –22], in particular machine learning methods. As an approximation of the competitiveness function, a polynomial of the n-th degree is taken, where n is the order of approximation. The problems of linear approximation and binary classification are solved, as a result of which the coefficients of the polynomial are determined.

The purpose of this work is to construct a competitiveness function for a model of a bilingual community, which takes into account: the effect of language acquisition by children at an early age; the different likelihood of adults acquiring a second language; the effect of mutual assistance within one language group (different for each language) [23–25]. The constructed competitiveness functions are used to predict the development of the dynamics of language competition in communities.

2. MATERIALS AND METHODS

2.1. Mathematical model

The principle of interaction between native speakers in a community generalizes the well-known hypothesis of effective meetings [26], which was used in the Volterra model. Let's take the following hypotheses to build the model:

— members of the community can speak one of two languages, conventionally called "first" and "second", or two at once; z_1 — the proportion of community members who speak only the first language, z_2 — the proportion of community members who speak only the second language, z_{12} — the proportion of community members who speak two languages (bilinguals);
— the proportion of community members who do not speak any language is negligible;
— the size of the community is constant over time (the number of births is equal to the number of deaths), $z_1 + z_2 + z_{12} = 1$;
— the size of any language group is non-negative: $0 \le z_1, z_2, z_{12} \le 1$;
— proficiency in a particular language does not affect the fertility rate / mortality rate, the coefficient r characterizes the rate of generational change;
— the probability of simultaneous (spontaneous) acquisition of two languages by an individual is negligible;
— bilingual children initially acquire a first or second language with probabilities c_1 and c_2, respectively; it is assumed that $c_1 > c_2$ [23, 24];
— within language groups there is a mutual assistance effect, which is determined by the coefficients: α_1 — for the first language and α_2 — for the second language;
— prestige coefficients for the first and second languages are equal to b_1 and b_2, respectively.

Taking into account the input data, we find that the distribution of languages in the community characterizes the state of the following system:

$$\begin{cases} \dot z_1 = c_1 r z_{12} - b_1 z_1 (z_1 + z_{12})^{\alpha_1}, \\ \dot z_2 = c_2 r z_{12} - b_2 z_2 (z_1 + z_{12})^{\alpha_2}, \\ \dot z_{12} = b_1 z_1 (z_2 + z_{12})^{\alpha_1} + b_2 z_2 (z_1 + z_{12})^{\alpha_2} - r z_{12}, \\ c_1 + c_2 = 1, \qquad 1 < \alpha_{1,2} < 2, \\ z_1 + z_2 + z_{12} = 1. \end{cases} \tag{1}$$

By expressing z_{12} in terms of z_1 and z_2, model (1) can be brought into a system on the plane:

$$\begin{cases} \dot z_1 = c_1 r (1 - z_1 - z_2) - b_1 z_1 (1 - z_1)^{\alpha_1}, \\ \dot z_2 = c_2 r (1 - z_1 - z_2) - b_2 z_2 (1 - z_2)^{\alpha_2}, \\ z_1 + z_2 \le 1. \end{cases} \tag{2}$$

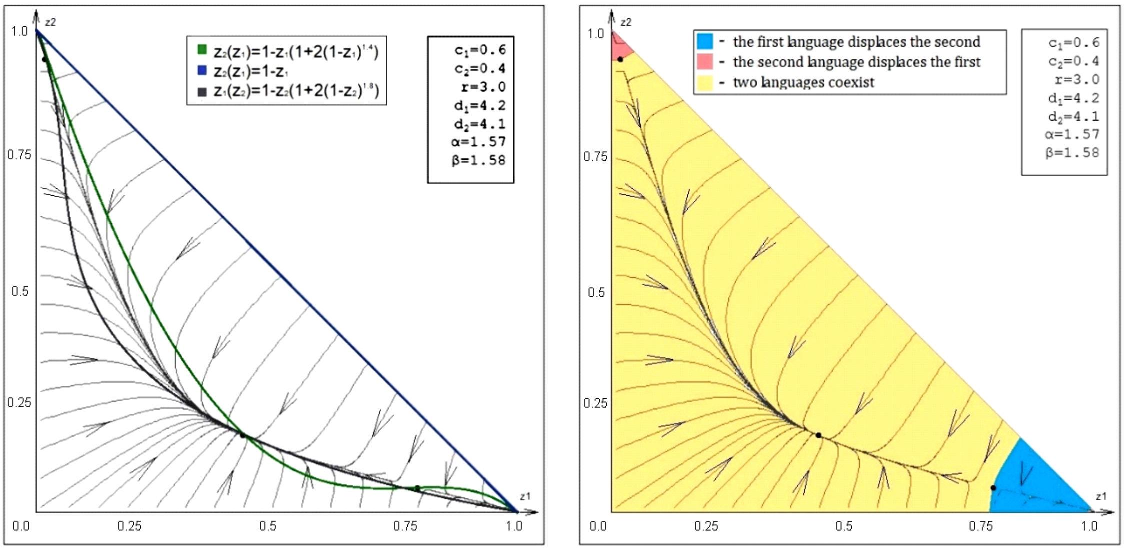
The equation for calculating the coordinate values of the singular points of the system (2) has the following form:

$$1 - \left(1 - z_1\left(1 + \frac{b_1}{c_1 r}(1 - z_1)^{\alpha_1}\right)\right)\left(1 + \frac{b_2}{c_2 r}\left(1 - \left(1 - z_1\left(1 + \frac{b_1}{c_1 r}(1 - z_1)^{\alpha_1}\right)\right)^{\alpha_2}\right)\right) = 0. \tag{3}$$

The leading term of equation (3) has $2\alpha_1 + \alpha_2 + 2$ degree. The number of special points is determined by the mutual assistance coefficients. The system (2) can have up to five equilibrium states, two of which are always at points $(0,1), (1,0)$ and are stable as a node. The solution to the system (2) shows two possible options for the development of dynamics:

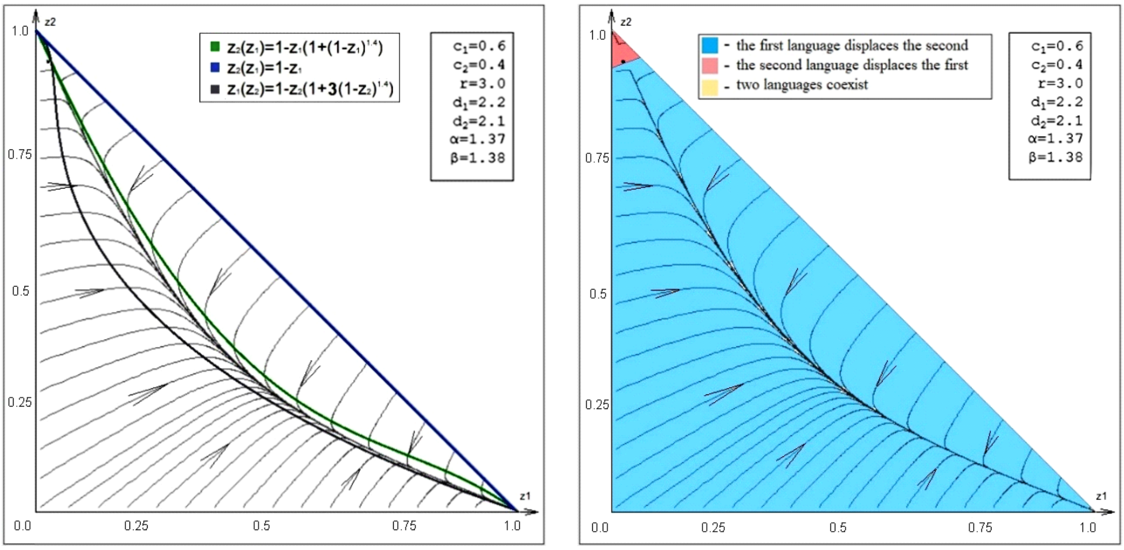
— coexistence of two languages and bilinguals: $z_1 > r$ or $z_2 > r$;
— displacement of one language by another.

Examples of phase portraits for (2) are given in the Annex (Fig. 1, 2).



(a) **(b)**

Figure 1. Phase portraits of the model (1) of the coexistence of two languages and bilinguals: *a)* isoclines of vertical and horizontal slop, *b)* zones of language dominance



(a) **(b)**

Figure 2. Phase portraits of the model (1) of the displacement of one language by another: *a)* isoclines of vertical and horizontal inclination, *b)* zones of language dominance

Stable states divide the phase space into basins of attraction in such a way that the phase trajectories of these basins eventually converge to one of them (Fig. 1, 2). Based on this, it can be argued that selection processes are carried out in this system. From the point of view of qualitative analysis of a dynamic system, selection is the movement of a phase trajectory towards a certain state of equilibrium located on the coordinate axis. In this regard, the following problem arises — how to determine which language will be preserved and which will be lost by the characteristics of the language. Sometimes the solution to this problem can be found in analytical form [27]. In our case, constructing this function in analytical form is extremely difficult. We propose a different approach to solve this problem. The competitiveness function (CF) is constructed as a function that depends on the parameters of the language: prestige; likelihood of language acquisition by children at an early age; mutual assistance parameter and the number of native speakers at a point in time.

2.2. Construction of the competitiveness function

Languages at any point in time are characterized by the following set of parameters: $v_1 = \{\alpha_1, c_1, b_1, z_{10}\}$ — for the first language, $v_2 = \{\alpha_2, c_2, b_2, z_{20}\}$ — for the second language.

Definition 1. *We will say that the i-th language is better than the j-th if the phase trajectory, with initial conditions determinedfrom v_i and v_j, eventually tends to the z_i coordinate axis, i.e. the z_j value will tend to zero, for $i, j \in \{1,2\}$.*

It is intended to express the CF as a power function of the following form:

$$
\begin{aligned}
J(z_{i0}, c_i, b_i, \alpha_i) = {} & \lambda_0 z_{i0} + \lambda_1 z_{i0} c_i + \lambda_2 z_{i0} \alpha_i + \lambda_3 z_{i0} b_i + \lambda_4 z_{i0} b_i c_i + \lambda_5 z_{i0} b_i \alpha_i + \\
& + \lambda_6 z_{i0} c_i \alpha_i + \lambda_7 z_{i0}^2 + \lambda_8 z_{i0}^2 c_i + \lambda_9 z_{i0} c_i^2 + \lambda_{10} z_{i0}^2 c_i^2 + \lambda_{11} z_{i0}^2 \alpha_i + \lambda_{12} z_{i0} \alpha_i^2 + \\
& + \lambda_{13} z_{i0}^2 \alpha_i^2 + \lambda_{14} z_{i0}^2 b_i + \lambda_{15} z_{i0} b_i^2 + \lambda_{16} z_{i0}^2 b_i^2 + \lambda_{17} z_{i0}^2 b_i c_i + \lambda_{18} z_{i0} b_i^2 c_i + \\
& + \lambda_{19} z_{i0} b_i c_i^2 + \lambda_{20} z_{i0} b_i^2 c_i^2 + \lambda_{21} z_{i0}^2 b_i c_i^2 + \lambda_{22} z_{i0}^2 b_i^2 c_i + \lambda_{23} z_{i0}^2 b_i \alpha_i + \lambda_{24} z_{i0} b_i^2 \alpha_i + \\
& + \lambda_{25} z_{i0} b_i \alpha_i^2 + \lambda_{26} z_{i0} b_i^2 \alpha_i^2 + \lambda_{27} z_{i0}^2 b_i \alpha_i^2 + \lambda_{28} z_{i0}^2 b_i^2 \alpha_i + \lambda_{29} z_{i0}^2 c_i \alpha_i + \lambda_{30} z_{i0} c_i^2 \alpha_i + \\
& + \lambda_{31} z_{i0} c_i \alpha_i^2 + \lambda_{32} z_{i0} c_i^2 \alpha_i^2 + \lambda_{33} z_{i0}^2 c_i \alpha_i^2 + \lambda_{34} z_{i0}^2 c_i^2 \alpha_i + \lambda_{35} z_{i0}^2 b_i^2 c_i^2 + \lambda_{36} z_{i0}^2 \alpha_i^2 c_i^2 + \\
& + \lambda_{37} z_{i0}^2 b_i^2 \alpha_i^2 + \lambda_{38} c_i + \lambda_{39} \alpha_i + \lambda_{40} b_i + \lambda_{41} b_i c_i + \lambda_{42} b_i \alpha_i + \lambda_{43} c_i \alpha_i + \lambda_{44} c_i^2 + \\
& + \lambda_{45} \alpha_i^2 + \lambda_{46} b_i^2 + \lambda_{47} b_i^2 c_i + \lambda_{48} b_i c_i^2 + \lambda_{49} b_i^2 c_i^2 + \lambda_{50} \alpha_i c_i^2 + \lambda_{51} c_i \alpha_i^2 + \lambda_{52} c_i^2 \alpha_i^2 + \\
& + \lambda_{53} b_i^2 \alpha_i + \lambda_{54} b_i \alpha_i^2 + \lambda_{55} b_i^2 \alpha_i^2 + \lambda_{56} b_i^2 c_1^2 \alpha_i^2 .
\end{aligned}
$$

Definition 2. *The $J(v_i)$ functional, which determines the order of advantage within the $\{v_i\}$ set, will be called a competitiveness function if from $J(v_i) > J(v_j)$ it should follow that the i-th language is preferable to the j-th at a time t.*

To determine the coefficients, we assign the following vector to each $v_{1,2}$:

$$
M(v_i) = \{z_{i0}, z_{i0} c_i, z_{i0} \alpha_i, z_{i0} b_i, z_{i0} b_i c_i, \ldots, b_i^2 \alpha_i^2, b_i^2 c_i^2 \alpha_i^2\}, \quad i \in \{1,2\},
$$

and to the pair (v_i, v_j) we assign a point $\big(M(v_i) - M(v_j)\big)$ and to the pair (v_j, v_i) — point $\big(M(v_j) - M(v_i)\big)$ in the 55-dimensional parameter space — $\{MV\}$. If the 1st language is better than the 2nd language, then the following inequality will be true:

$$
J(v_1) > J(v_2),
$$

and if the 2nd language is better than the 1st language, then the inequality will be reversed.

This inequality can be expanded as follows:

$$
\begin{aligned}
& \lambda_0 z_{10} + \lambda_1 z_{10} c_1 + \lambda_2 z_{10} \alpha_1 + \lambda_3 z_{10} \alpha_1 c_1 + \ldots + \lambda_{54} \alpha_1^2 b_1 + \lambda_{55} b_1^2 \alpha_1^2 + \lambda_{56} b_1^2 c_1^2 \alpha_1^2 > \\
& \quad > \lambda_0 z_{20} + \lambda_1 z_{20} c_2 + \lambda_2 z_{20} \alpha_2 + \lambda_3 z_{20} \alpha_2 c_2 + \ldots + \lambda_{54} \alpha_2^2 b_2 + \lambda_{55} b_2^2 \alpha_2^2 + \lambda_{56} b_2^2 c_2^2 \alpha_2^2 .
\end{aligned}
$$

After cancellation we obtain an inequality defining the area of the $\{MV\}$ space which contains all the $M(v_i)$ points at which the 1st language is better than the 2nd language:

$$\lambda_0(z_{10}-z_{20})+\lambda_1(z_{10}c_1-z_{20}c_2)+\lambda_2(z_{10}\alpha_1-z_{20}\alpha_2)+\lambda_3(z_{10}\alpha_1c_1-z_{20}\alpha_2c_2)+\dots$$
$$+\lambda_{54}(\alpha_1^2b_1-\alpha_2^2b_2)+\lambda_{55}(b_1^2\alpha_1^2-b_2^2\alpha_2^2)+\lambda_{56}(b_1^2c_1^2\alpha_1^2-b_2^2c_2^2\alpha_2^2)>0. \quad (4)$$

If expression (2) is turned into equality, then the hyperplane equation will be obtained, which will divide the $\{MV\}$ space into two parts. In one of them, the 1st language will be better than the 2nd language, and in the other, vice versa. Thus, the construction of the competitiveness function is reduced to determining the normal of the hyperplane, which is specified by the vector of the λ_i coefficients:

$$\bar{N}=(\lambda_0,\lambda_1,\lambda_2,\dots,\lambda_{56}).$$

The CF formula is obtained as a power function. By substituting the language parameters into the CF, we obtain the values of its competitiveness. By comparing the resulting competitiveness of languages, we can say that over time, the language that has the greatest competitiveness will remain. The boundary dividing the phase space into basins of attraction will pass through the points at which the CF values for the first and second languages are equal.

3. MATERIALS AND METHODS

3.1. Construction of a training sample

On the phase plane, 1500 arbitrary points were selected with different initial conditions for the languages from which the sets $v_1=\{\alpha_1,c_1,b_1,z_{10}\}$ and $v_2=\{\alpha_2,c_2,b_2,z_{20}\}$ were formed. An example of such samples is shown in the figure (Fig. 3, 4).

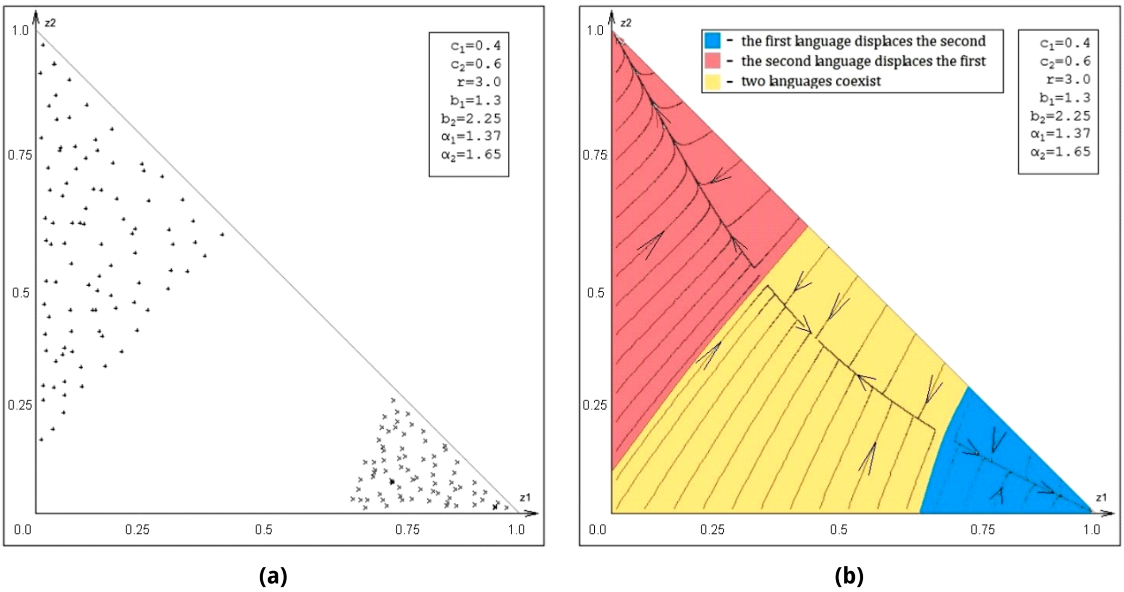
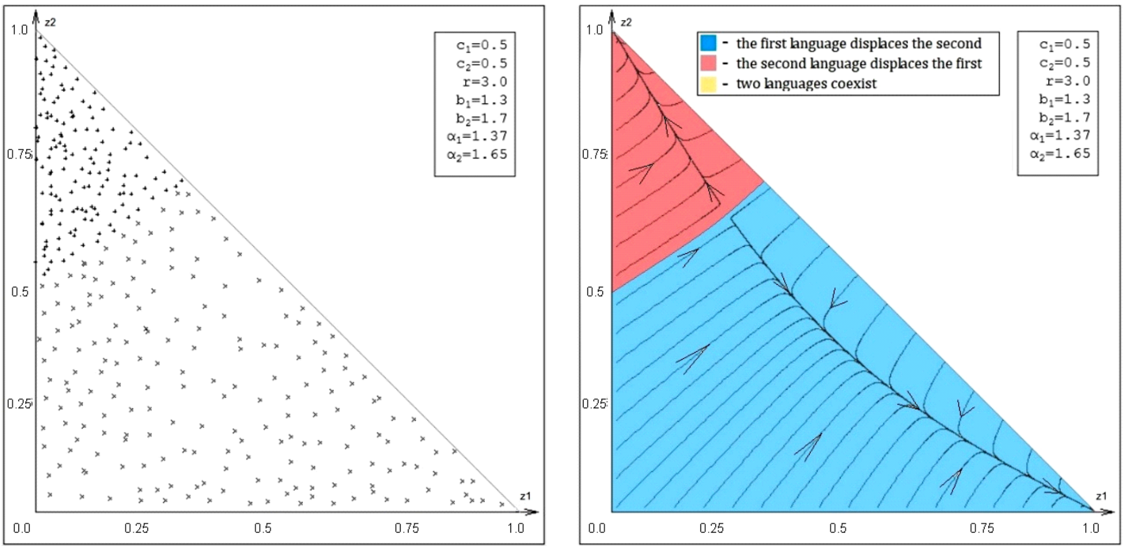


Figure 3. *a)* fragments of v_1 and v_2 training samples: crosses — the first language dominates, pros — the second one dominates, *b)* language dominance



(a) **(b)**

Figure 4. *a)* fragments of v_1 and v_2 training samples: crosses — the first language dominates, pros — the second one dominates, *b)* language dominance

For various parameters of languages $\{c_1, c_2, \alpha_1, \alpha_2, b_1, b_2\}$, by solving a system of differential equations, it was established which language would win as a result of competition for all v_1 and v_2 sets. The set of points $\big(M(v_1) - M(v_2)\big)$ и $\big(M(v_2) - M(v_1)\big)$ was calculated. This set of points was classified according to the survival of one of the two languages. Using the method of support vectors, the dividing line equation for these two classes was obtained. From the equation of the dividing line, the values of the λ_i coefficients were obtained. The results of the procedures for restoring the coefficients of the competitiveness function are presented in Table 1.

Table 1. Coefficients of the competitiveness function

λ_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9
0.68871	0.33907	1.97972	1.05904	0.51731	3.03885	0.98361	0.55978	0.27119	0.16886
λ_{10}	λ_{11}	λ_{12}	λ_{13}	λ_{14}	λ_{15}	λ_{16}	λ_{17}	λ_{18}	λ_{19}
0.13298	1.57116	6.00722	4.68234	0.89468	1.75330	1.54712	0.4303	0.84849	0.25561
λ_{20}	λ_{21}	λ_{22}	λ_{23}	λ_{24}	λ_{25}	λ_{26}	λ_{27}	λ_{28}	λ_{29}
0.41532	0.20941	0.73823	0.35628	0.77055	0.49302	3.00151	0.38146	2.31434	1.51036
λ_{30}	λ_{31}	λ_{32}	λ_{33}	λ_{34}	λ_{35}	λ_{36}	λ_{37}	λ_{38}	λ_{39}
1.15252	2.52058	5.02047	9.17206	4.37632	7.49758	15.05785	12.99112	0.0008	0.10667
λ_{40}	λ_{41}	λ_{42}	λ_{43}	λ_{44}	λ_{45}	λ_{46}	λ_{47}	λ_{48}	λ_{49}
–0.16	–0.07669	–0.39787	0.0675	0.0008	0.61227	–0.576	–0.27849	–0.03677	–0.13475
λ_{50}	λ_{51}	λ_{52}	λ_{53}	λ_{54}	λ_{55}	λ_{56}			
0.0411	0.37037	0.21883	–1.75125	–1.00585	–5.45174	–1.14723			

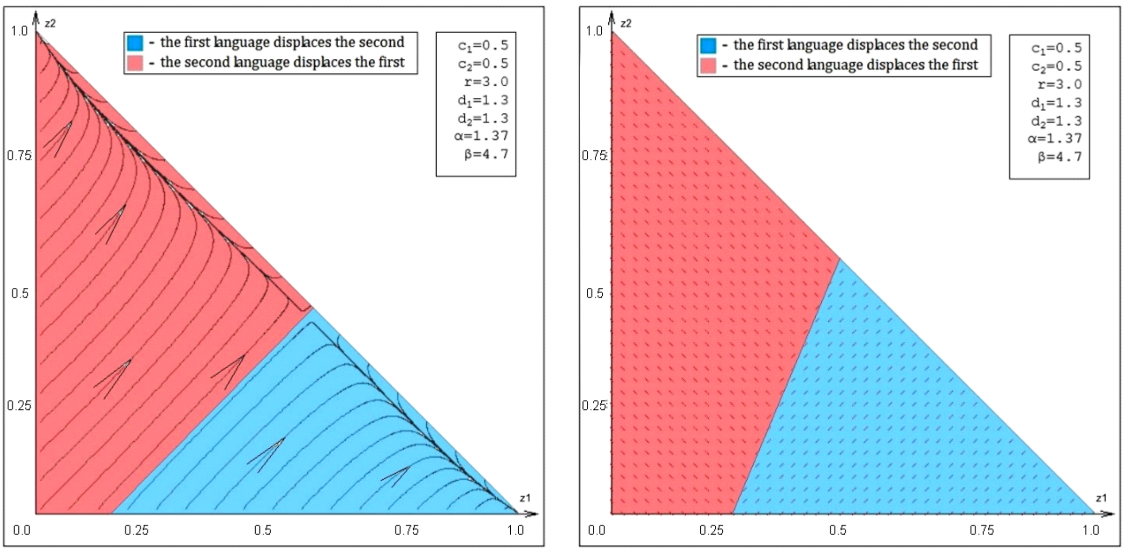
Competitiveness function:

$$
\begin{aligned}
J(z_{i0}, c_i, b_i, \alpha_i) = {} & 0.68871 z_{i0} + 0.33907 z_{i0} c_i + 1.97972 z_{i0} \alpha_i + 1.05904 z_{i0} b_i + \\
& + 0.51731 z_{i0} b_i c_i + 3.03885 z_{i0} b_i \alpha_i + 0.98361 z_{i0} c_i \alpha_i + 0.55978 z_{i0}^2 + \\
& + 0.27119 z_{i0}^2 c_i + 0.16886 z_{i0} c_i^2 + 0.13298 z_{i0}^2 c_i^2 + 1.57116 z_{i0}^2 \alpha_i + \\
& + 6.00722 z_{i0} \alpha_i^2 + 4.68234 z_{i0}^2 \alpha_i^2 + 0.89468 z_{i0}^2 b_i + 1.75330 z_{i0} b_i^2 + \\
& + 1.54712 z_{i0}^2 b_i^2 + 0.4303 z_{i0}^2 b_i c_i + 0.84849 z_{i0} b_i^2 c_i + 0.25561 z_{i0} b_i c_i^2 + \\
& + 0.41532 z_{i0} b_i^2 c_i^2 + 0.20941 z_{i0}^2 b_i c_i^2 + 0.73823 z_{i0}^2 b_i^2 c_i + 0.35628 z_{i0}^2 b_i \alpha_i + \\
& + 0.77055 z_{i0} b_i^2 \alpha_i + 0.49302 z_{i0} b_i \alpha_i^2 + 3.00151 z_{i0} b_i^2 \alpha_i^2 + 0.38146_{i0}^2 b_i \alpha_i^2 + \\
& + 2.31434 z_{i0}^2 b_i^2 \alpha_i + 1.51036 z_{i0}^2 c_i \alpha_i + 1.15252 z_{i0} c_i^2 \alpha_i + 2.52058 z_{i0} c_i \alpha_i^2 + \\
& + 5.02047 z_{i0} c_i^2 \alpha_i^2 + 9.17206 z_{i0}^2 c_i \alpha_i^2 + 4.37632 z_{i0}^2 c_i^2 \alpha_i + 7.49758 z_{i0}^2 b_i^2 c_i^2 + \\
& + 15.05785 z_{i0}^2 \alpha_i^2 c_i^2 + 12.99112 z_{i0}^2 b_i^2 \alpha_i^2 + 0.0008 c_i + 0.10667 \alpha_i - 0.16 b_i - \\
& - 0.07669 b_i c_i - 0.39787 b_i \alpha_i + 0.0675 c_i \alpha_i + 0.0008 c_i^2 + 0.61227 \alpha_i^2 - \\
& - 0.576 b_i^2 - 0.27849 b_i^2 c_i - 0.03677 b_i c_i^2 - 0.13475 b_i^2 c_i^2 + 0.0411 \alpha_i c_i^2 + \\
& + 0.37037 c_i \alpha_i^2 + 0.21883 c_i^2 \alpha_i^2 - 1.75125 b_i^2 \alpha_i - 1.00585 b_i \alpha_i^2 - \\
& - 5.45174 b_i^2 \alpha_i^2 - 1.14723 b_i^2 c_1^2 \alpha_i^2.
\end{aligned}
$$

3.2. Restoring the competitiveness function

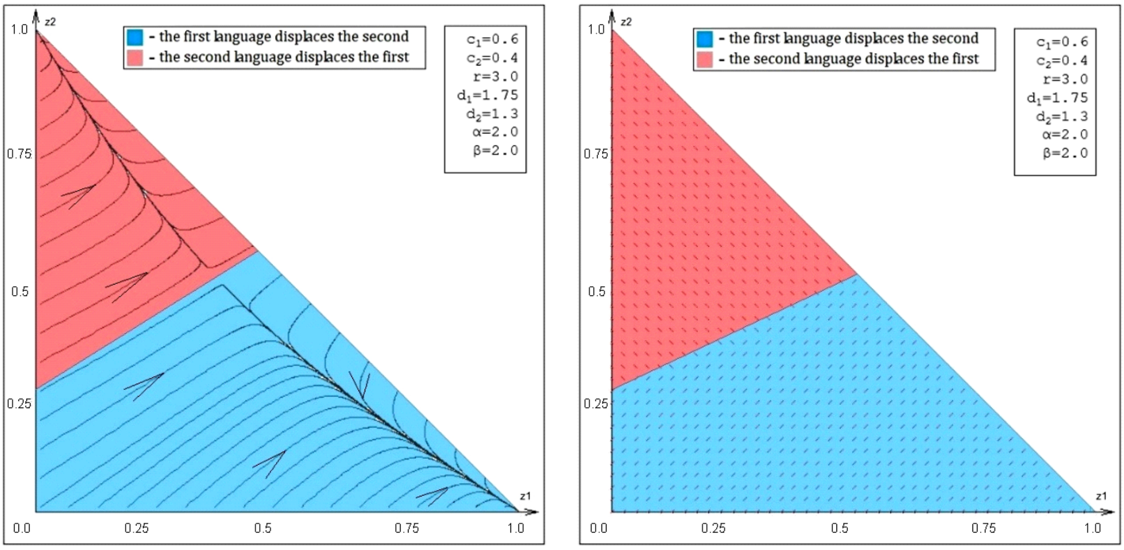
The resulting CF formula was tested for random parameters of the model (1) and showed a good prediction result (Fig. 5, 6). Graphs of the CF (Fig. 7, 8).

Statistical data on the shares of Welsh and English languages for 1901–2001, as well as Gaelic and English for 1891–1971 in England are considered [26, 27]. Using the least squares method, the parameters of model (1) were identified; the values of the coefficients are given in Table 2.



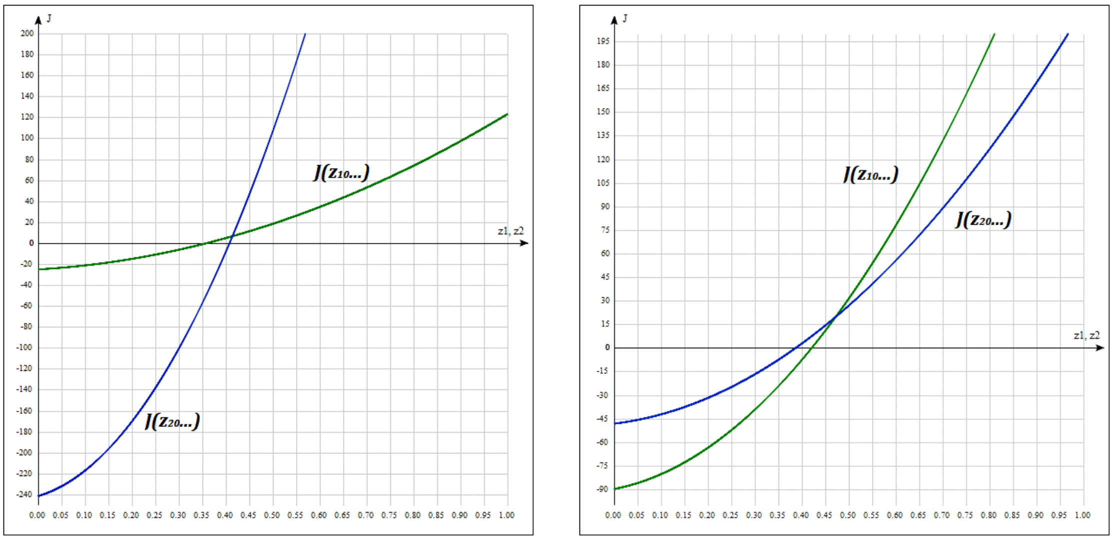
(a) (b)

Figure 5. Areas of language dominance obtained as a result of: *a)* integration, *b)* CF analysis



(a) (b)

Figure 6. Areas of language dominance obtained as a result of: *a)* integration, *b)* CF analysi



(a) (b)

Figure 7. FCS graphs: *a)* for model parameters in (Fig. 5), *b)* for the model parameters in (Fig. 6)

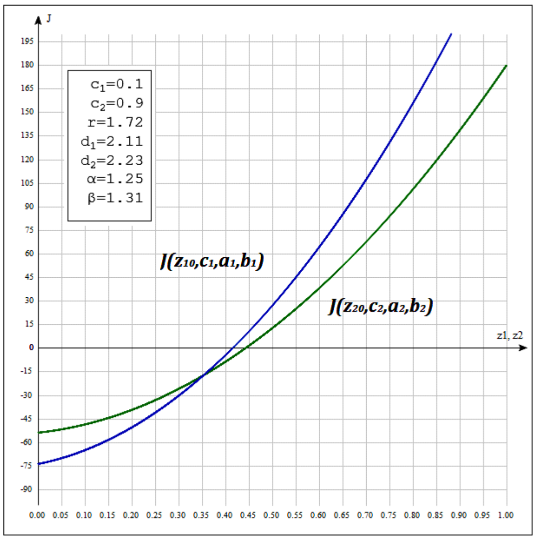
Table 2. Model coefficients

Language group	b_1	b_2	c_1	c_2	α_1	α_2	r
Welsh-English	2.11	2.23	0.1	0.9	1.25	1.31	1.72
Gaelic-English	5.44	5.43	0.1	0.9	2.52	2.54	1.41

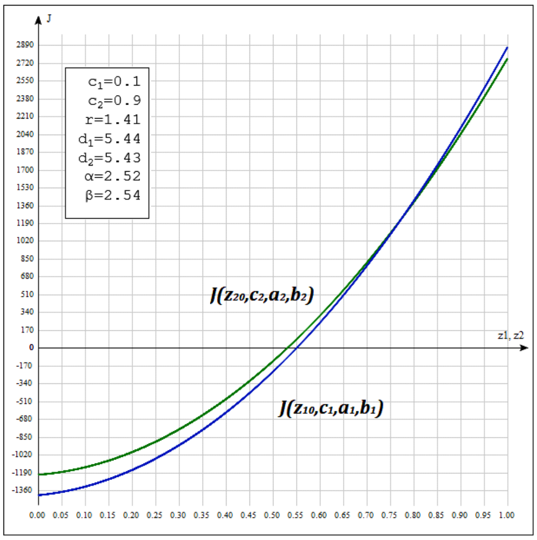
Based on the data given in the Table 1, competitiveness formulas were obtained for language pairs from the Table 2. Graphs of the competitiveness functions are shown in (Fig. 8), CF formulas are given in Table 3.

Table 3. Language competitiveness functions

Welsh and English	
CF of the Welsh language	$J(z_{10}, c_1, \alpha_1, b_1) = 32.12566z_1 + 201.72475z_1^2 - 54.00443$
CF of the English language	$J(z_{20}, c_2, \alpha_2, b_2) = 59.27785z_2 + 284.35816z_2^2 - 73.91503$
Gaelic and English	
CF of the Gaelic language	$J(z_{10}, c_1, \alpha_1, b_1) = 356.77187z_1 + 3617.03874z_1^2 - 1211.91169$
CF of the English language	$J(z_{20}, c_2, \alpha_2, b_2) = 433.86036z_2 + 3843.03588z_2^2 - 1406.94469$



(a)



(b)

Figure 8. CF graphs: *a)* for the Welsh-English language pair, *b)* for the Gaelic-English language pair

The CF values for the Welsh and English languages as of 2001 were calculated, as well as the CF values for the Gaelic and English languages as of 1971 were calculated (Table 4).

Table 4. The values of the competitiveness function

CF values as of 2001	
CF of the Welsh language	0.0
CF of the English language	1779288.312
CF values as of 1971	
CF of the Gaelic language	0.0
CF of the English language	28458998.415

4. CONCLUSION

The CF values allow us to conclude that in the future, under unchanged conditions, English will be dominant, and the number of Welsh and Gaelic languages in bilingual communities will decrease to zero over time. The forecast constructed as a result of modeling is in good agreement with the statistical data considered in the work. Comparing the CF values for different initial parameters is actually equivalent to determining the basins of attraction in the model. If the number of languages in the considered community is large enough (an example of such a community may be the Internet), then solving the research problem using classical methods becomes cumbersome and very complex. In this case, an algorithm based on maximizing the competitiveness function seems to be a convenient alternative to the classical approach. Using the constructed competitiveness function, it is possible to model language dynamics by numerically solving the minimax problem. The proposed methodology for constructing the competitiveness function is quite general and can be applied to a wide range of models of language dynamics.

In the work, a new model of a bilingual community is constructed and investigated, in which languages differ in the following parameters: the prestige of the language; the likelihood of language acquisition at an early age; coefficient of mutual assistance within one language group; initial number of native speakers. A variant of dynamics with the effect of mutual assistance of speakers of the same language to each other is considered, which is similar to the effect of language volatility in the Abrams-Strogatti model. An analytical study based on the mathematical theory of selection was carried out. The search and construction of the competitiveness function was carried out. The result was applied to statistical data. The technique was tested on a model with an analytical solution. The adequacy and effectiveness of this method for constructing the CF on a model taking into account various language volatility and the variable number of native speakers was demonstrated. Forecasts on language dynamics were obtained.

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Функция конкурентоспособности для модели двуязычного сообщества

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Аннотация

Цель настоящей работы состоит в построении функций конкурентоспособности для модели двуязычного сообщества. Материалы и методы. В работе используется новая модель двуязычного сообщества, в которой учитываются: эффект освоения второго языка в раннем возрасте; эффект взаимопомощи внутри группы одного языка. В модели языки характеризуются параметрами престижности, вероятностью освоения языка в раннем возрасте, параметром взаимопомощи и начальным количеством носителей языка. Рассматривается задача определения результатов конкуренции языков по их характеристическим параметрам. Результаты. Предлагается новая методика решения задачи о результатах языковой конкуренции. Для этого в языковой динамике вводятся новое понятие: функция конкурентоспособности. Для восстановления функции конкурентоспособности применяется метод ранжирования, который сводится к разделению упорядоченных пар языков (при фиксированных начальных условиях) на два класса «первый язык вытесняет второй» и «второй язык вытесняет первый». Функция конкурентоспособности ищется в виде степенной функции, зависящей от параметров языка. При этом осуществляется идентификация значений коэффициентов функции на основе обработки имеющихся данных о динамике модели. Производится анализ значений функций конкурентоспособности, сравнение результатов с наблюдаемой статистикой и на этой основе строится прогноз дальней динамики развития. Применение данной методики демонстрируется на модели, в которой поиск решения в аналитическом виде является затруднительным. Заключение. Предложенная методика построения функции конкурентоспособности является достаточно общей и вполне может быть применена для широкого круга моделей описывающих динамику популяций. Прогноз, составленный на основе построенных функций конкурентоспособности, хорошо согласуется с эмпирическими данными.

Ключевые слова: *языковая конкуренция, языковая динамика, билингвизм, двуязычие, отбор, сохранение языка, функция конкурентоспособности, критерий отбора, процессы отбора, математическая модель, обыкновенные дифференциальные уравнения.*

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