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USING TECHNOLOGY WITH FUTURE TEACHERS OF PRIMARY SCHOOL MATHEMATICS: A GLIMPSE INTO AN AMERICAN EXPERIENCE

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Abstract

The paper describes the author's experience in using technology within mathematics content and methods (undergraduate and graduate) courses for prospective teachers of primary grades (age 5–10). The main pedagogical idea behind the courses is to change pre-teachers' perception of mathematics as a subject matter most people predictably dislike. It is suggested that technology can assist instructors in making mathematics an enjoyable subject matter without sacrificing content. The paper provides examples of using Excel, *Wolfram Alpha*, dynamic geometry software, computer graphing program, and the Online Encyclopedia of Integer Sequences. In conclusion, solicited comments by teacher candidates about their experience of learning computer assisted mathematics are shared.

Keywords: *mathematics, teacher education, primary grades, Excel, Wolfram Alpha, Geometer's Sketchpad, the Graphing Calculator, OEIS®.*

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1. INTRODUCTION

Lillard in her book about Maria Montessori put it very nicely: "education would be more successful were it not so frequently disliked" [15, p. 4]. This quote is truly applicable to mathematics teacher education. Among different voices of elementary teacher candidates, even at the graduate level, the following confession is pretty common when describing their mathematical experience: *"Math has never been my number one subject, and as such taking multiple math courses at the graduate level made me very nervous."* In a much stronger voice, another teacher candidate shared: *"As I speak with many of my fellow graduate students and soon to be student teachers, many of us agree that we do not love math!"*. Given such sentiments about mathematics, the task of mathematical educators is to design mathematics courses to make mathematics a likable subject matter among elementary teacher candidates. This paper will show a way to use commonly available tools of technology and some real-life contexts to make mathematics a more likable discipline. And most importantly, to make it likable and enjoyable not at the expense of compromising on the subject matter training as such a compromise often leads to "negative effects on instructional quality and student progress" [7].

In the United States, elementary mathematics teacher preparation may be a responsibility of the School of Arts and Sciences hosting the department of mathematics or of the School of

Education hosting the department of elementary education. Not very often both departments share the responsibility of preparing teachers to teach mathematics as there might be little common interest and shared knowledge between the members of the two schools. More than half a century ago, a memorandum regarding the development of pre-college mathematics curriculum was published [3] in which the differences between professional educators and mathematicians were described as follows: "Mathematicians, reacting to the dominance of education by professional educators who may have stressed pedagogy at the expense of content, may now stress content at the expense of pedagogy and be equally ineffective." [3, p. 189]. A notable example of collaboration between mathematicians and mathematics educators is the keynote address by Benoit Mandelbrot at the 7th International Congress on Mathematical Education in which he recognized a challenge for teachers to navigate between joyful and requisite aspects of mathematics learning: "Motivate the students by that which is fascinating and hope that the resulting enthusiasm will create sufficient momentum to move them through that which is no fun but is necessary" [16, p. 86]. Perhaps the use of the word fascinating by the creator of fractal geometry was due to the visual effects of the Mandelbrot Set which, since its first appearance in 1980, "has assumed iconic status, conquering the world's computer screens in the role of the ultimate screen-saver" [18, p. 117]. Put another way, in the modern-day classroom, the use of technology and the teaching of mathematics go hand in hand, whatever the student population and their grade level. In the context of mathematics teacher preparation at the primary level, the problem of instruction is to appropriately link the fascinating content of informational and computational power of computers to mathematically unsophisticated yet pedagogically intricate content of primary school mathematics. As mentioned by Beckmann in the general context of mathematics teacher preparation, "examples of surprisingly intricate details that are involved in understanding elementary math are everywhere" [8, p. 370].

In the author's experience working for different groups, mathematicians and educators, a typical mathematics faculty member does not have much interest in teaching mathematics to prospective elementary teachers. It is indeed difficult to be excited about teaching when your students struggle with fractions, even in the most basic situations. As one mathematician shared with the author, once he asked an undergraduate student to multiply 7/11 by 11/7. The student immediately took a calculator, spent an hour, but failed to provide an answer. The use of a simple calculator is indeed not very helpful for this task, especially when one can confuse arithmetical keys of a calculator in the darkness of a room (like young children often do when adding, say, 2 and 3, but hit the division key and become curious, at best, about the result they see). How can one teach fractions (or mathematics, in general) to elementary teacher candidates as an engaging, computer assisted subject matter? This paper demonstrates several applications of the modern-day technology tools integrated with informal and friendly contexts. In conclusion, teacher candidates' solicited reflections on those efforts are shared.

2. FROM PIZZAS TO EGYPTIAN FRACTIONS

Pizza is a very popular meal in the United States. Circular pizzas are sold in square-shaped boxes. The shape of such a box prompts intuitive understanding of how to cut pizza in two or four equal parts by driving a pizza wheel from corner to corner of the box. Less intuitively clear is how to divide a circular pizza in five equal pieces. (For that, a pizza shaped as a rectangle would be more appropriate). Yet, circular pizzas are commonly used as a context to teach fractions beyond 1/2 and 1/4. This makes most students confused — when asked to show 1/3 or 1/5 of a circle, they cut it in three or five not necessarily equal parts and see one out of three or five such parts as 1/3 or 1/5, respectively. That is where technology can come into play. Using the *Geometer's Sketchpad* [14], one of the first dynamic geometry programs in the world, one can create computer images of unit fractions that can be used to model the division of pizzas into equal pieces. Such use of technology supports conceptual understanding of a fraction because, according to Gestalt psychology, "many phenomena of experience are variations organized around Prägnanzstufen, phases of clear-cut structure ... [so] that an angle of 93° is not seen as an entity in its own right but as a "bad" right angle" [5, p. 183]. In particular, this explains difficulties in using the geometric approach when developing a computer program capable of recognizing cursive handwriting with same letters having different geometric descriptions due to their deviations from Prägnanzstufen [13].

Wolfram Alpha is a computational knowledge engine available free online. *Wolfram Alpha Pro* is available by subscription only. This more advanced version provides many computational details not available without subscription. Teacher candidates have to be aware of the existence of *Wolfram Alpha Pro* because it can do homework for students by providing step-by-step solutions to many problems from the primary school curriculum (see in the conclusion comments by teacher candidates about using *Wolfram Alpha* with their own children). This raises the issue of how one can modify traditional problems to make them technology immune in a sense that such a problem cannot be solved by technology alone. At the same time, the importance of technology in the teaching of mathematics calls for problems to be also technology enabled; that is, the use of technology significantly assists one in solving a problem. This issue is discussed in detail elsewhere [2].

As an example, consider a problem which uses fair division of pizzas among several individuals as a problem-solving context. *How can one divide four pizzas among five people fairly having fewer than 20 pieces?* To answer this question, teacher candidates are introduced to an Egyptian fraction — a finite sum of different unit fractions. Entering in the input box of *Wolfram Alpha* the command "4/5 as Egyptian" yields the sum $1/2 + 1/4 + 1/20$ (Figure 1) generated by the program apparently using the Greedy algorithm. That is, the task posed above is technology enabled (not in a sense of using a calculator to multiply 7/11 by 11/7). Whereas such representation can be obtained through the Greedy algorithm (explained to the candidates later, once they are epistemically at ease with fractions) by pencil and paper, this would be a challenge for a typical elementary teacher candidate at that point. A technology immune part of this task is to interpret the (computer-generated) Egyptian fraction in terms of the division of pizzas, demonstrate such division on a (self-drawn) picture, count the number of pizza pieces, and understand that this number, 15, can be obtained without drawing a picture. Alternatively, a picture can be created in the context of the *Geometer's Sketchpad* (Figure 2) using a special program written by the instructor [1]. An important aspect of this activity is that the division of the quarter of a pizza into five equal parts is a combination of drawing and mathematics; that is, through drawings informed by technology one recognizes in a drawn piece the fraction 1/20. That is, by putting together several unit fractions, to experience the creation of another unit fraction, an elementary teacher candidate develops an experimental understanding of the connection between numeric and geometric domains. It is at this point that a question whether there exist unit fractions which cannot be iterated to create another unit fraction unexpectedly can be perceived by an elementary teacher candidate as making sense.

Regarding the above task involving pizzas, *Wolfram Alpha*, and the *Geometer's Sketchpad* the following classic quote is worth mentioning with the goal to be reformulated in the context of teaching fractions. "Although at an early stage of mathematical development, quantitative reasoning and arithmetical thinking of a child are pretty vague and immature in comparison with

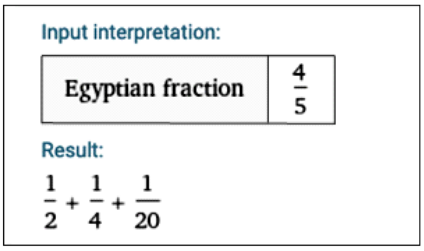


Figure 1. Using *Wolfram Alpha*

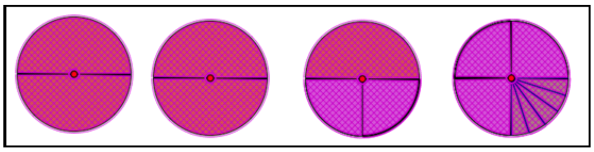


Figure 2. Using the *Geometer's Sketchpad*

those of an adult with whom the child interacts, it is through this interaction that the final forms of reasoning and thinking about numbers, that have to be developed as a result of having an adult in his/her environment, are somehow present at that stage and, not only present but in fact define and guide the child's first steps toward the development of the final forms of understanding quantity and comprehending arithmetic" [19, p. 84, translated from Russian by the author]. Replacing a child by an elementary teacher candidate and an adult by a mathematics instructor, one can argue that through the use of *Wolfram Alpha* as a generator of Egyptian fractions and the *Geometer's Sketchpad* as milieu for partitioning pizzas informed by those fractions, the final forms of reasoning and thinking about integers and fractions burgeon inside the candidate's *experimental* understanding of the properties of numbers and guide such understanding to much higher levels of comprehending arithmetic. As far as the role of geometric representations of the arithmetic of fractions goes, long before Vygotskian educational theory and the modern-day digital tools, the importance of visual thinking as a foundation of conceptual understanding was emphasized by a Swiss reformer of education Johann Heinrich Pestalozzi (1746–1827) who, according to [5, p. 299], "forced the children to draw angles, rectangles, lines, and arches, which he said constituted the alphabet of the shape of objects, just as letters are the elements of the words." Using the dynamic geometry software in the construction of fractions and seeing a rather complex (for an elementary teacher candidate) Egyptian fraction generated by *Wolfram Alpha* in response to a non-complicated, yet professionally formulated request, provide a friendly milieu within which final forms of comprehending properties of integers and relationships among fractions develop. A possible sense making of the question about the existence of unit fractions that cannot be iterated to create another unit fraction is an example of one's move from a joy of operating electronic fraction circles to the foundation of number theory "which is no fun but is necessary" [16, p. 86] for elementary teacher candidates to "know how the mathematics they teach is connected with that of prior and later grades" [10, p. 1].

To conclude this section, consider the question: *How can one divide three pizzas among seven people fairly having fewer than 21 pieces?* Using *Wolfram Alpha* to answer this question by generating an Egyptian fraction representation of $3/7$, with many other cases typically resulting in fewer cases than the straightforward division of each pizza into seven pieces, may present a challenge as the program yields $3/7 = 1/3 + 1/11 + 1/231$ implying that exactly 21 pizza pieces result when division is informed by the last Egyptian fraction. The same number results from dividing each pizza into seven pieces. This not only shows elementary teacher candidates the complexity of mathematics (much different from multiplying $7/11$ by $11/7$), but it motivates the introduction of another fair division of pizzas shown in Figure 3. Generalizing from this figure allows one to say that m (identical) pizzas can be divided fairly among n people, $n > m$, by having $m(n–m)$ pieces measured by $1/n$ of a pizza and m pieces measured by m/n of a pizza. As shown in Figure 3, when $m = 3$ and $n = 7$ we have 15 pieces (rather than 21). Likewise, when $m = 4$ and

$n = 5$ we have 8 pieces (rather that 15 provided by the Egyptian fraction). The algebraic form of this alternative division makes it possible to model numerically the behavior of the number of pizza pieces as a function of n and m, $n > m$. Educationally, such computerization of pizza pieces serves two purposes: to show the power of generalization in the application of mathematics to real life and to emphasize the complexity of mathematics behind Egyptian fractions which do not allow similar modeling through algebraic generalization.

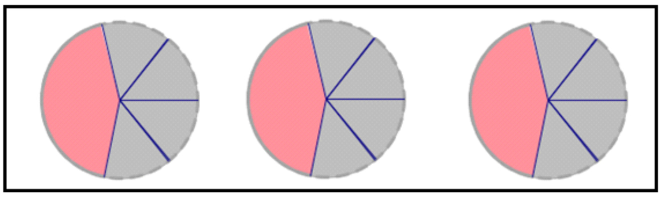
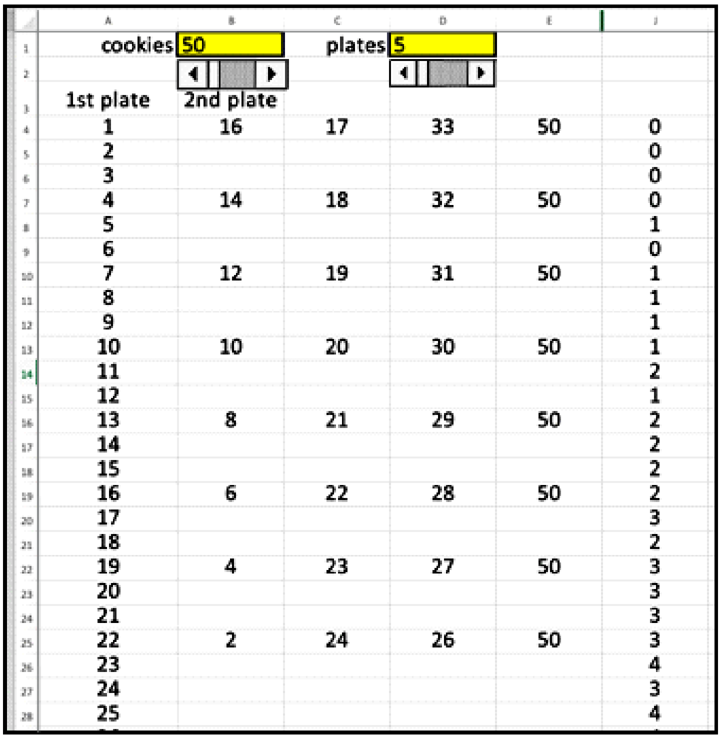


Figure 3. Dividing 3 pizzas among 7 people in 15 pieces

3. FROM COOKIES AND FIBONACCI-LIKE NUMBERS TO OEIS® VIA SPREADSHEETS

Cookie is another popular food item in the United States. All integer arithmetic can be taught in the context of cookies. Being physically divisible (not as easy as pizzas though), their context can also be used to open a window into fractions. Furthermore, the appropriate use of this problem-solving context enables mathematics instructors to go beyond the primary school arithmetic and, in doing so, to use technology. As an opening illustration, consider the following problem: *Having four plates filled with cookies and ten cookies on the fourth plate, how many cookies are there on the first two plates, so that each plate beginning from the third plate has as many cookies as the previous two plates combined?* The discussion of this problem, which has 'more than one correct answer', eventually leads to the introduction of Fibonacci and Fibonacci-like numbers followed by the construction of a spreadsheet towards the end of changing the number of plates, the number of cookies on the last plate, and, given the two numbers, recording the number of ways to put cookies on the first two plates. For example, the spreadsheet pictured in Figure 4 shows eight ways to put cookies on the first two plates to have 50 cookies on the 5th plate. Algebraically, setting x and y to represent, respectively, the number of cookies on the first two plates, each plate having at least one cookie, the first five Fibonacci-like numbers $x, y, x+y, x+2y, 2x+3y$ can be formed so that the variation of the fifth number shows how different boxes with two or three cookies in a box may be created. A special spreadsheet technique of using a formula with the so-called circular reference (i.e., referencing the cell where the formula is defined) makes it possible to generate a sequence (Figure 4, column F) the terms of which show the number of ways to put cookies on the first two plates when the number of cookies on the fifth plate varies. One can see that the sequence 1, 0, 1, 1, 1, 1, 2, 1, 2, 2, 2, 2, 3, 2, 3, 3, 3, 3, ... represents the number of ways to put cookies on the first two plates so that the fifth plate has n cookies, $n \le 5$. Once such a sequence is recorded by the spreadsheet, the next step is to familiarize teacher candidates with the Online Encyclopedia of Integer Sequences (OEIS®) — a rich source of information most of which, however, is far beyond the primary school level. Pedagogically, introducing the OEIS® to elementary teacher candidates makes it possible to discuss how in the digital age, with an easy access to the Internet, one can learn to manage the abundance of information provided [14]. Learning to manage unfamiliar data provided by technology and to extract relevant information from it is an important skill that elementary teacher candidates need to bring with them into their own computer assisted classroom. The OEIS® includes the above sequence under the number

A103221 with many complex mathematical interpretations one of which may look familiar: it is the number of partitions of a positive integer greater than 4 in parts 2 and 3 (each of which is a non-zero part). A teacher candidate would experience self-esteem (and even joy) knowing that sequence A103221 has a (possibly unknown) situational reference in terms of cookies on plates and Fibonacci-like numbers. It is through the appropriate use of technology that a mathematics instructor can bridge cookie-based explorations with formal mathematics to enable the development of interest in and the decrease of anxiety and dislike regarding the subject matter among elementary teacher candidates.



A	B	C	D	E	J
cookies	50	plates	5		
1st plate	2nd plate				
1	16	17	33	50	0
2					0
3					0
4	14	18	32	50	0
5					1
6					0
7	12	19	31	50	1
8					1
9					1
10	10	20	30	50	1
11					2
12					1
13	8	21	29	50	2
14					2
15					2
16	6	22	28	50	2
17					3
18					2
19	4	23	27	50	3
20					3
21					3
22	2	24	26	50	3
23					4
24					3
25					4

Figure 4. There are eight ways of ending up with 50 cookies on the 5th plate

4. REPRESENTING COOKIES AND PLATES IN COMPUTER GRAPHING SOFTWARE

As was mentioned in Section 2 of this paper, elementary teacher candidates need experience in connecting mathematics they teach to what their students were taught in the past and would be taught in the future [10]. With this in mind, the candidates can also be introduced to graphic representations of the results of numeric computations made possible by the use of a spreadsheet. To this end, moving towards more formal mathematics, note that among eight positive integer solutions of the equation

$$2x+3y=50, \tag{1}$$

where x and y stand for the number of cookies on the first and the second plates, respectively, two solutions generate the largest and the smallest values for the fourth Fibonacci-like number $x+2y$, namely, 33 and 26. Using the *Graphing Calculator* produced by Pacific Tech [6], a computer program capable of graphing two-variable equations and inequalities, one can graph simultaneously in the (x, y)–plane a straight line defined by equation (1) and a region defined by the inequality $x+2y<n$ with n being a slider-controlled parameter. One can see (Figure 5) that when $n=33$ (the number of cookies on the 4th plate) the region and the line intersect at the point with the coordinates $(1,16)$. Likewise, one can see (Figure 6) that when $n=26$ (the number of cookies on the 4th plate) such point of intersection has the coordinates $(22,2)$. The results can be seen within the spreadsheet of Figure 4.

Alternatively, one can use *Wolfram Alpha* (Figure 7) to solve simultaneously equation (1) and the equation

$$x+2y=n, \tag{2}$$

with parameter n to obtain the solution $x=100–3n$, $y=2(n–25)$. Elementary teacher candidates, being computer assisted in finding this algebraic solution, are capable, nonetheless, by using trial and error, to conclude that the largest value of n for which x is a positive integer is $n=33$ and the smallest value of n for which y is a positive integer is $n=26$. That is, $n=33$ yields $(x,y)=(1,16)$ and $n=26$ yields $(x,y)=(22,2)$. These pairs of numbers, found independently by Excel, the *Graphing Calculator*, and *Wolfram Alpha*, are exactly the quantities of cookies put on the first two plates to have 50 cookies on the 5th plate[1]. In the words of Freudenthal, “it is independency of new experiments that enhances credibility” [12, p. 193] of computer assisted problem solving.

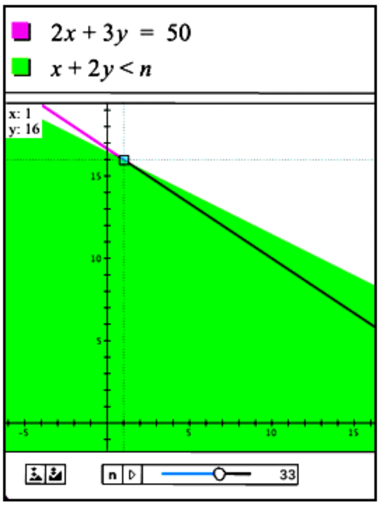


Figure 5. 33 cookies on the 4th plate

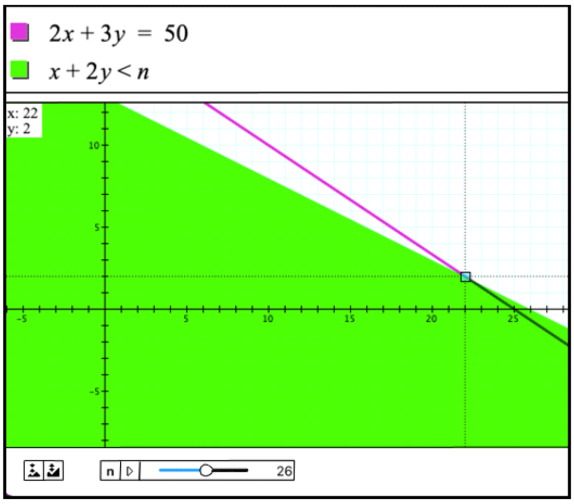


Figure 6. 26 cookies on the 4th plate

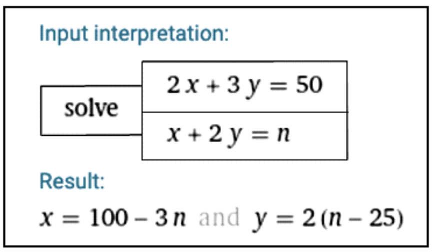


Figure 7. Solving a system of two linear equations with a parameter

[1] As a way of demonstrating conceptual intricacy of mathematics to elementary teacher candidates, one can use *Wolfram Alpha* or the *Graphing Calculator* and expand the product $(x^2+x^4+x^6+\ldots)(x^3+x^6+x^9+\ldots)$ to see that the coefficient in x^{50} coincides with the number of positive integer solutions of equation (1). Alternatively, the relations $x^{50}=x^2x^{48}=x^8x^{42}=x^{14}x^{36}=x^{20}x^{30}=x^{26}x^{24}=x^{32}x^{18}=x^{38}x^{12}=x^{44}x^6$ describe eight ways of putting 50 cookies in boxes with two or three cookies in a box.

5. PRACTICING FORMAL REASONING IN THE CONTEXT OF NUMERIC MODELING

As mentioned by the Conference Board of the Mathematical Sciences — an umbrella organization of nineteen professional societies in the United States, concerned, in particular, with mathematics teacher preparation — "mathematics courses that explore elementary school mathematics in depth can be genuinely college-level intellectual experiences, which can be interesting for instructors to teach and for teachers to take" [10, p. 31]. Indeed, another approach to finding two positive integers, the largest and the smallest, assumed by the sum $x+2y$ while enabling equation (1) to hold true, can be such an experience for elementary teacher candidates. The approach, beginning from reasoning by trial-and-error, provides the candidates with much needed practice in formal reasoning that can be implemented in computationally simple yet conceptually intricate context of addition and subtraction.

More specifically, starting from 50, one can check to see if it can be preceded by 49. Algebraically, assuming that equation (1) is satisfied for a pair (x, y), one has to check whether this pair satisfies equation (2) when $n = 49$. If that is case, then 49 is preceded by 1 and 1 must be preceded by 48 and this is impossible as no positive integer may be added to 48 to get 1. In other words, the 4$^{\text{th}}$ plate may not have 49 cookies. Similarly, one can check to see whether 50 can be preceded by, say, 20. In that case, 20 must be preceded by 30 and this is not possible as no positive integer may be added to 30 to get 20.

This reasoning is similar to the method of infinite descent or to the "well-ordering principle" [4, p. 34] when, in the set of positive integers, a precedes b if and only if b is ether equal to a or is the sum of a and some positive integer. Obviously, 49 and 20 do not satisfy this principle when $b = 50$. With this in mind, teacher candidates may be asked to try to find the largest and the smallest number of cookies that the 4$^{\text{th}}$ plate has when the 5$^{\text{th}}$ plate has 50 cookies. The candidates must be helped to recognize that if 50 is preceded by a number smaller than 26, then the next number is greater than 24 and already descending from 50 to 25 requires the next descent to 25 and then to zero, something that contradicts the well-ordering principle. Likewise, if 50 is preceded by a number greater than 33 then the next number is smaller than 17 and already descending from 50 to 34 requires the next descent to 16 followed by 18, something that contradicts the well-ordering principle. This practice in formal reasoning is very helpful for it gives elementary teacher candidates much needed experience in preparing their future students to be "[m]athematically proficient" [9, p. 8].

6. CONCLUSION THROUGH THE VOICES OF TEACHER CANDIDATES

This paper started with quoting a few disappointing comments by elementary teacher candidates about mathematics, a subject matter they have to teach. However, as computer assisted mathematical courses, the candidates were enrolled in, drew to a close, their comments about mathematics have changed. Some of such (solicited) comments are worth sharing.

In the words of one teacher candidate, *"The course gave us the tools and much of the knowledge to allow us to see mathematics in a different way which will help us open our students' eyes to the world of mathematics. Fractions have always been a difficult topic to learn in past experiences and have recently been a challenge to teach to my practicum students. I will now be able to use the Wolfram Alpha website when teaching my students as well as to demonstrate to students how to use it whether that be at school or at home".* It appears that by seeing mathematics in a different way, the candidate acknowledges a departure from the tradition to dislike it. Many elementary teacher candidates enter the course mentioned in this comment seeing mathematics from the rule method perspective [17]. This outdated method, being grounded in pure memorization followed by drill and practice enhanced nowadays by the use of a calculator or by various computer programs in which encouraging praises like "good job" in response to the right answer are geared towards students' liking mathematics. This change, however, is not the case. Whereas such praises are absent in *Wolfram Alpha*, the program played a positive role in the author's effort to have students *"see mathematics in a different way"*. Surprisingly, as follows from the next comment, the program is not known to practicing teachers.

"My experience in using Wolfram Alpha has been a very positive experience, not only for helping me to better understand how to learn and teach fractions, but to help me with other mathematical problems as well. I have actually used this website to assist my 9-year-old daughter and 10-year-old son with some math problems they have brought home for homework. I also shared this website with their teachers. Their teachers had never heard of it and they were very impressed as well. I liked the website so much that I paid for the subscription". The teacher candidate's comment spans across multiple matters acknowledging the usefulness of *Wolfram Alpha* within three major mathematical education contexts: pre-service, in-service, and family. Thus, computer assisted mathematics teacher education came full circle by connecting pre-service with in-service via family. Indeed, many teacher candidates, especially at the graduate level, are non-traditional students with their own children of the primary school age. One such non-traditional student shared with the author, *"This class has taught me ways of thinking about math which I hadn't really touched on in other classes. I have even used some of the methods in helping my children with their math homework at night and have seen success in using them, which is very exciting!"* This comment might be a result of the appropriate use of cookies and spreadsheets as an invigorating enhancement of traditional school mathematics. If this excitement about teaching traditionally dislikable things would reach future students of elementary teacher candidates to allow the former to like those things, then mathematics teacher education may be considered as a genuinely successful enterprise.

Another teacher candidate was referring to multiple technology tools encountered in a mathematics education course. *"In my experience in this course your use of computer tools such as spreadsheets, Wolfram Alpha, and the Geometer's Sketchpad to explain mathematical concepts has given me the opportunity to visualize mathematical concepts with ease. The use of technology to learn math is efficient way to use an alternative method to come to the answer of a question and to conceptualize mathematical topics."* This comment most likely reflects the candidate's experience using *Wolfram Alpha* and dynamic geometry software in the context of sharing pizzas and *Wolfram Alpha* and a spreadsheet in the context of cookies on plates. In particular, the comment suggests that computer assisted mathematics can show teacher candidates a friendly face of fractions. It points at the candidate's seeing technology as an assistant in conceptualization rather than a means of memorization. This is not to say that memorization is a bad thing. But the appropriate use of computers allows for conceptualization to precede memorization so that the latter develops kind of involuntary. For example, almost intuitive understanding that three pizzas for five people is a better deal than for six people, enhanced by the visual power of electronic fraction circles, inadvertently commits to memory a grip that 1/2 is the largest unit fraction smaller than 3/5, something that is the first step in applying the Greedy algorithm to the latter fraction. This position is reflected in the comment of another future elementary teacher: *"Wolfram Alpha helped me as an educator to come to understand theoretical concepts and answers to very unobtainable problems in a matter of seconds. When completing the Egyptian fractions assignment, I did quite appreciate the visual aids that often were used. The more and more we dove into it, the more fractions seemed to click in my head. I found that overall, the practice and the visual aids really did a majority of the heavy lifting in my growing ability to learn fractions".*

To conclude, note that the comment *"I love the idea of using technology in the classroom"* shared by yet another elementary teacher candidate is quite straightforward and emboldening for educational technology enthusiasts. Unfortunately, among the candidates' comments, the author could not find such direct expression of love towards mathematics to match against the two comments mentioned in the introduction. But the use of digital tools in an elementary mathematics methods and content course allowing future teachers of mathematics *"to see mathematics in a different way"*, *"to visualize mathematical concepts with ease"*, and *"to come to understand theoretical concepts and answers to very unobtainable problems in a matter of seconds"* is a testament to the success of computer assisted mathematics as a teaching method enabling primary school classrooms to be led by teachers who, at the very least, do not dislike teaching fundamentals of arithmetic to young children and do not need a calculator to multiply 7/11 by 11/7.

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Использование технологий будущими учителями математики начальной школы: взгляд на американский опыт

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Аннотация

В статье описывается опыт автора по использованию технологий в содержательно-методических курсах по математике (бакалавриат и магистратура) для будущих учителей начальных классов (возраст 5–10 лет). Основная педагогическая идея курсов состоит в том, чтобы изменить восприятие математики учителями начальной школы как предмета, который, как ожидается, не нравится большинству людей. Предполагается, что технологии могут помочь преподавателям сделать математику интересным предметом без ущерба для содержания. В статье приведены примеры использования Excel, *Wolfram Alpha*, программного обеспечения для динамической геометрии, программы компьютерного построения графиков и онлайн-энциклопедии целочисленных последовательностей. В заключение приводятся письменные комментарии будущих учителей об их опыте изучения математики с помощью компьютера.

Ключевые слова: *математика, педагогическое образование, начальные классы, Excel, Wolfram Alpha, Geometer's Sketchpad, графический калькулятор, OEIS®.*

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